Semiempirical Analytical Model for the Spin Modulation of Retarding Potential Analyzer Fluxes

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The spin modulation of ion fluxes to a spinning retarding potential analyzer (RPA) has been computed numerically from a theoretical model that has been found to represent satellite observations well in the forward direction. These calculations cover ion Mach numbers from 0.2 to 4.2, normalized (dimensionless) spacecraft potentials from −5 to +5, and spin angles from 0 to 90 deg. A relatively simple empirical, analytical expression has been developed to represent these curves very accurately as functions of these three parameters. Over the range of parameters investigated, maximum deviations of the model from the original values are less than 16%, these occurring for spin angles of 30 and 80 deg, while average values are within 4%. The spin modulation is found to be more sensitive to ion Mach number than to spacecraft potential, but with sensitivity increasing as the spacecraft potential becomes more positive. This latter characteristic makes the spin modulation a useful supplement to the customary energy analysis of RPA ion data, particularly for determining spacecraft potential for a positively charged spacecraft.

Nomenclature

A_i	= ith polynomial fit coefficient, $i = 0,\ldots,3$
$B_{i,j}$	= jth polynomial fit coefficient to ith variable, $i,j = 0,\ldots,3$
$C_{i,j,k}$	= kth polynomial fit coefficient to the i,jth variable, $i,j,k = 0,\ldots,3$
E	= ion kinetic energy
e	= absolute value of unit electronic charge
F	= flux of ions into a retarding potential analyzer
F_D	= thin sheath approximation to F, where the detector axis is in the ram direction
$F_D(\theta)$	= same as F_D with M replaced by $M\cos\theta$
F_0	= zero-order approximation to F_D for $M,L = 0$
$f(\theta)$	= arbitrary function of θ
I_0	= modified Bessel function of order zero
k	= Boltzmann's constant
L	= square root of sum of V_0 and V_G
M	= ionic Mach number
m_i	= ion mass
N	= ion number density
P_n	= polynomial of order n
T_i	= ion temperature
V_0	= spacecraft potential normalized to kT_i
V_G	= RPA grid potential normalized to kT_i
V_s	= spacecraft velocity
α_0	= angle a particle trajectory makes with the detector axis, at the detector
α_∞	= same angle, at large distances
ϕ_s	= spacecraft potential
θ	= spin phase angle of detector axis relative to ram direction
θ_p	= half-angle acceptance of an equivalent circular aperture for a detector

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Introduction

THE modulation of unretarded ion fluxes to a retarding potential analyzer (RPA) on a spinning spacecraft moving through a space plasma can provide important information about the ambient plasma properties and about the electrical potential of the spacecraft. To extract this information, however, requires a detailed knowledge of how this spin modulation responds to the different significant parameters. A mathematical model of this response has been provided by Whipple et al.[1] in the form of an integral expression. This integral has been evaluated analytically by Comfort et al.[2] for the "ram" (spacecraft velocity) direction by means of a "thin-sheath" approximation. However, even with this approximation, it has not been possible to evaluate the integral analytically for an arbitrary spin or ram angle (angle measured with respect to the ram direction). A quantitative numerical analysis of the variations of this expression with several parameters, comparing the thin sheath and Coulomb potential limits, has also been carried out by Young and Farrugia.[3]

Numerical evaluation of the integral expression of Whipple et al.[1] is a time-consuming process on anything less than a CRAY-class computer, which is not a desirable system to employ for routine data analysis. Therefore, it is useful to develop analytical expressions to represent the spin modulations of ion fluxes in order to extract the information contained in them in a timely and convenient manner during the analysis of large volumes of RPA data. The purpose of this paper is to describe how such expressions can be derived, to provide examples for a specific instrument [the retarding ion mass spectrometer (RIMS) on Dynamics Explorer 1 (DE-1)], to assess their accuracy, and to examine how the spin modulation varies with spacecraft potential and Mach number.

Technique and Results

Since it has been demonstrated that the equation of Whipple et al.[1] represents the spin modulation of the RIMS observations within 90 deg of the ram direction,[4] the present task is to represent that equation in a form that is both accurate and usable (i.e., rapidly computable). In order for an empirical expression to be accurate, it must have a form that will adequately represent the proposed variation. To be feasible, it must have a form that can be readily fitted to the "data." Our approach is first to establish a data set to which we fit the expression. Then we determine a form for the empirical expression that will meet these criteria. Finally, we develop a method for computing the desired fluxes, given the input variables.

In this case, our data set consists of a catalog of spin curves obtained by numerically integrating Eq. (18) of Whipple et al.[1] for a range of Mach numbers and spacecraft potentials covering the anticipated range of observations and an appropriate range of spin phase angles. (This equation was derived in the context of plasmaspheric observations, and arguments were made for the validity of the assumptions used in the derivation for these conditions.) This equation is given by

$$F = \iint (E - V_0) \exp(-E - M^2 + 2ME^{1/2} \cos\theta \cos\alpha_\infty) I_0(2ME^{1/2} \sin\theta \sin\alpha_\infty)\, dE\, d(\sin^2\alpha_\infty) \quad (1)$$

In evaluating this integral numerically, we have used the thin-sheath limit in all cases and have limited consideration to the front side of the satellite, that is, to spin phase angles from 0 to 90 deg at 5-deg intervals. The range of ion Mach numbers, given by

$$M = V_s / \sqrt{(2kT_i/m_i)} \quad (2)$$

is from 0.2 to 4.2 in increments of 0.4; this adequately spans all RIMS observations of $H+$ and $He+$ and most observations of $O+$ in the vicinity of the plasmasphere[1,4] (see also the range of Mach numbers in Table 3 of Samir et al.[6]). For spacecraft potential, a dimensionless variable is used, defined by

$$V_0 = e\phi_s / kT_i \quad (3)$$

Typical thermal ion temperatures in and near the plasmasphere range from about 0.2 to a few eV. Spacecraft potentials in this region typically range from -1 V to 3 or 4 V for analyzable data. The lowest temperatures occur with the most negative potentials (at lowest altitudes) and vice versa. So we have taken the range for V_0 to be -5.0 to $+5.0$ in increments of 1.0, which should be adequate for RIMS observations in and near the plasmasphere.[4,6] The total number of spin curves in this catalog is 121, representing 11 values of Mach number and 11 values of spacecraft potential. The total number of points evaluated is 2299, requiring approximately 35 h of CPU time on a CRAY XMP416 computer. For comparison, analysis of 60 min of RIMS data to determine ion temperatures and densities typically requires evaluation of a spin curve more than 4000 times, using present fitting procedures.

In seeking an expression to represent the spin modulation accurately, we have tried two different approaches. The first is to begin with the analytical model for the RPA curve in the thin-sheath approximation, which was derived by Comfort et al.[2] for the ram direction. If we could use this as a basis for an empirical fit to the spin modulation, we would have a complete model for both spin and RPA variations. The basic equation for the ram direction is given by[2]

$$F_D = F_0 (\exp[-(L - M)^2] + \sqrt{\pi} M [1 - \mathrm{erf}(L - M)] - \cos^2\theta_p \exp(V_0 \tan^2\theta_p - M^2 \sin^2\theta_p) \{\exp[-(L \sec\theta_p - M \cos\theta_p)^2] + \sqrt{\pi}\, M \cos\theta_p [1 - \mathrm{erf}(L \sec\theta_p - M \cos\theta_p)]\}) \quad (4)$$

where $F_0 \equiv N(kT_i/2\pi m_i)^{1/2}$, and $L^2 \equiv V_0 + V_G$. Here we attempt to incorporate the spin modulations by including a $\cos\theta$ factor with each Mach number, as has been done with some of the neutral planar approximations.[5] The results have been totally inadequate and not worth pursuing.

Table 1 Mathematical models tested

Function	Sum of mean square deviations[a]	Maximum fractional deviation	Spin angle of maximum deviation
$\exp[P_1(\theta^2)]$	8.31	1.23	0
$F_D(\theta)^*\exp[P_1(\theta^2)]$	4.82	1.44	70
$F_D(\theta)^*\exp[P_3(\theta)]$	1.86	0.422	15
$\exp\{P_3[\cos(\theta)]\}$	0.084	0.0959	90

[a]Summed over entire set of original curve-fits.

The second approach to determining an appropriate form for the analytical expression is to use a direct fit to a simple function that seems to have the qualitatively correct variation with the spin phase angle θ. These are basically variations on an expression of the form

$$F = F_0 \exp[f(\theta)]$$

The specific models tried for $f(\theta)$ are listed in Table 1. The results of these fits have been tested by comparing sums of the mean square deviations of the fitted curves from the originals, these sums extending over the entire catalog of spin curves. By this criterion, the function which best represents the spin modulation is clearly the exponential of the third-order polynomial in $\cos\theta$. So we adopt the following as our working expression:

$$F_D(\theta) = F_0 \exp(A_0 + A_1 \cos\theta + A_2 \cos^2\theta + A_3 \cos^3\theta) \quad (5)$$

To determine the Mach number M and normalized spacecraft potential V_0 dependence in terms of this expression, we first fit each spin curve to this functional form. This results in four coefficients for each of the 121 spin curves. The spin curves are then grouped into 11 groups of constant V_0, each containing 11 spin curves, corresponding to 11 different Mach numbers. Corresponding coefficients A_i within each group (four sets of 11 in each of 11 groups) are fit to third-order polynomials in M. (Second-order fits do not give sufficient accuracy, and other simple functions do not fit the variation.) The results are 16 coefficients B_{ij} of the form

$$A_0 = B_{00} + B_{01}M + B_{02}M^2 + B_{03}M^3$$
$$\vdots$$
$$A_3 = B_{30} + B_{31}M + B_{32}M^2 + B_{33}M^3 \quad (6)$$

for each of the 11 sets of curves at constant V_0.

Next, corresponding coefficients B_{ij} from each of these 11 sets are fit to third-order expressions in V_0, resulting in the following expressions for each of the 16 coefficients in the above equations:

$$B_{ij} = C_{ij0} + C_{ij1} V_0 + C_{ij2} V_0^2 + C_{ij3} V_0^3 \quad (7)$$

This gives a total of 64 coefficients C_{ijk} which can then be used in reverse order, given values of M and V_0, to compute successive coefficients and finally to evaluate the normalized flux. These coefficients are tabulated in Table 2. Note that RIMS is comprised of three different sensors, one radial head, with effective equivalent circular aperture half-angle 30 deg, and two identical end heads, with equivalent circular aperture half-angle 47 deg. Since the radial head is the spinning detector, Table 2 is based on a 30-deg half-angle aperture.

To test the accuracy of this overall procedure (the cumulative effects of three sets of curve fits), the resulting expressions were used to regenerate the data set curves; these new curves

Table 2 Fit coefficients for 30-deg aperture

		C_{ijk}			
	j	$k=1$	$k=2$	$k=3$	$k=4$
$i=1$	1	−2.1333E−2	1.5976E−2	−1.4964E−4	−9.9570E−5
	2	−2.5794E+0	−5.5022E−1	−3.5967E−2	6.6872E−3
	3	−1.2919E−1	9.5935E−2	1.2113E−2	−2.5142E−3
	4	−5.5219E−2	−4.9506E−3	−1.0541E−3	2.5395E−4
$i=2$	1	−2.6281E−2	1.2244E−2	−2.4274E−3	6.5298E−5
	2	2.7413E+0	4.1865E−1	4.5724E−2	−5.9046E−3
	3	−1.0133E−2	6.5469E−2	−7.6564E−3	−6.0740E−4
	4	6.5487E−2	−1.6425E−2	−3.9859E−4	3.3172E−4
$i=3$	1	3.3814E−1	−6.4385E−2	−8.9619E−4	7.3791E−4
	2	−1.2467E+0	2.9546E−1	7.0885E−3	−4.3585E−3
	3	8.8165E−1	−3.1139E−1	−2.2453E−2	6.8684E−3
	4	6.5832E−2	4.2064E−2	4.5447E−3	−1.1839E−3
$i=4$	1	−2.8718E−1	3.6297E−2	3.4255E−3	−7.1599E−4
	2	1.0729E+0	−1.6383E−1	−1.6672E−2	3.6129E−3
	3	−7.3449E−1	−1.4930E−1	−1.7875E−2	3.7647E−3
	4	−7.6511E−2	−2.0416E−2	−3.0750E−3	5.9948E−4

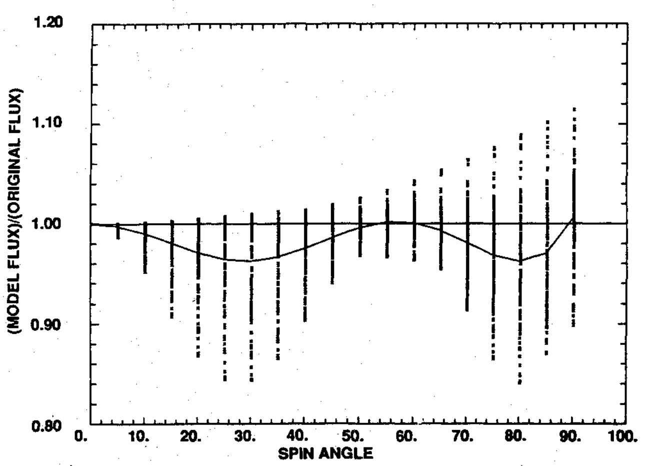

Fig. 1 Ratios of model flux to original flux as functions of spin angle, at 5-deg intervals; each point for a given angle represents a different combination of Mach number and normalized potential.

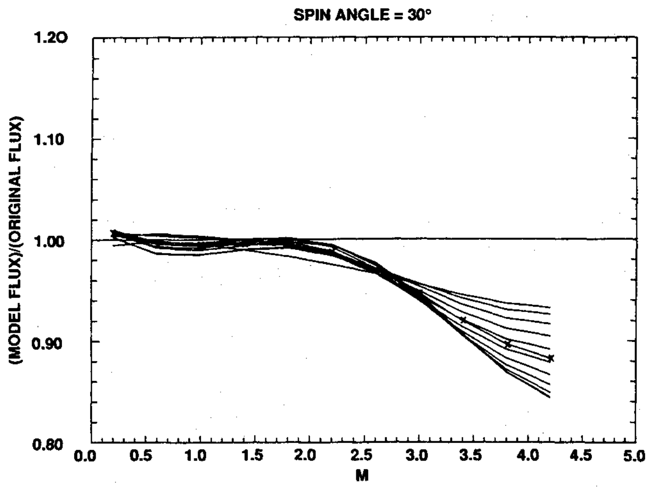

Fig. 2a Ratios of model flux to original flux as functions of Mach number at 30-deg spin angle; solid lines connect points for the same normalized potential.

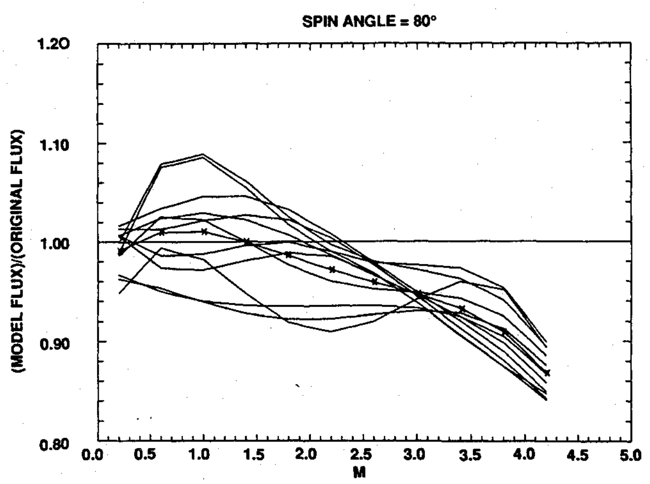

Fig. 2b Same as Fig. 2a, but for 80-deg spin angle. At M = 4.2, the curves correspond to potentials which generally decrease from top to bottom; at other potentials the behavior is less systematic.

were then compared with the original curves. In Fig. 1 we plot the ratio of the model curve to the original curve, as a function of spin phase angle for the entire catalog of curves. We are primarily concerned with "maximum differences" since these place limits on the accuracy of the analysis and could result in a systematic skewing of results in a data analysis program. From Fig. 1 we see that maximum fractional deviations between original spin curves and those resulting from our empirical expressions are about +0.11 and −0.16, the latter occurring for angles near 30 and 80 deg. Considering the parameter range covered, this is a very accurate representation. The continuous curve in this figure connects the average values at each angle; this curve remains within 4% of 1.0 throughout the range of spin angles.

To see the region of parameter space where the inaccuracies are greatest, we look at only spin phase angles of 30 and 80 deg and plot fractional deviation vs Mach number, connecting points for constant values of V_0, in Figs. 2a and 2b. The curves with "x" designating comparison points are the average curves. The contrast between the behavior at 30 deg and that at 80 deg is striking. Thirty-deg fractional deviations from 1.0 remain clustered with magnitudes less than about 0.04 out to $M \sim 2.5$, where all ratios become systematically lower than 1.0 with increasing M, out to the maximum deviation of −0.15. For 80 deg, however, significant deviations occur for $M < 1$ in both positive and negative directions; then for $M >$ 2.5, all exhibit a similar increasingly negative deviation. From Figs. 1 and 2, we see that if the empirical curves are fit to data within ± 60 deg of the ram direction, accuracies of the model curves will be better than 5% for Mach numbers less than 2.5. This would include virtually all of the plasmaspheric $H+$ and $He+$ observations from DE-1/RIMS.

We can look at the same fractional deviations from Fig. 1 as functions of normalized potential, instead of Mach number; these are shown in Figs. 3a and 3b for 30 and 80 deg, respectively. For 30 deg there is very little variation with normalized potential. Those curves clustered near 1.0 correspond to the low Mach numbers, whereas those progressively lower correspond to increasingly higher M, as we would expect from Fig. 2a. In Fig. 3b, most of the curves for constant M exhibit an oscillatory behavior, with no particular potential displaying consistently better accuracy than the others. Not surprisingly, given Fig. 2b, the curves showing the consistently lowest behavior correspond to the highest Mach numbers.

The above figures tend to magnify the differences, which was done to allow us to see more clearly where the inaccuracies tended to maximize in parameter space. To provide a more reasonable perspective, in Fig. 4 we plot the full spin curves, both original and those from our empirical model, for several cases where the deviations are largest, as found above. When

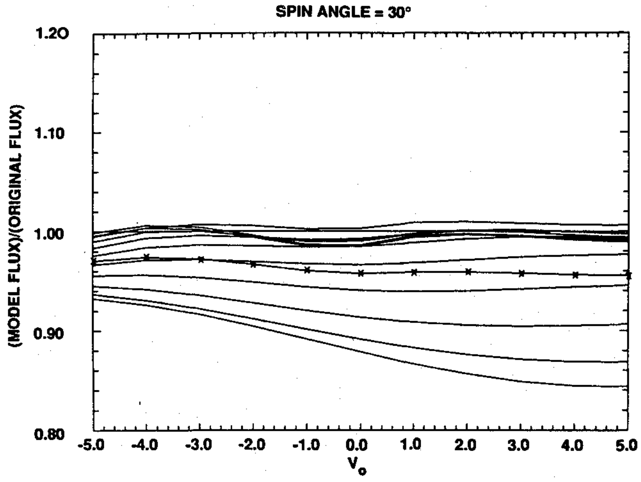

Fig. 3a Ratios of model flux to original flux as functions of normalized potential at 30-deg spin angle; solid lines connect points for the same Mach number.

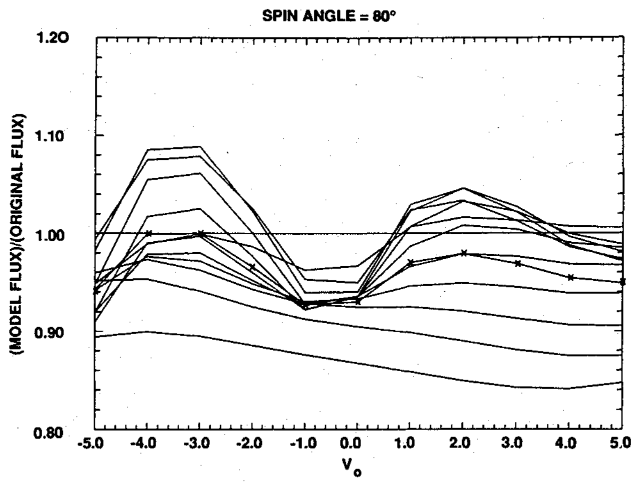

Fig. 3b Same as Fig. 3a, but for 80-deg spin angle. At $V_0 = 5$, the bottom curve is for $M = 4.2$, and M decreases monotonically (intervals of 0.4) for successively higher curves, up to the cluster near 0.98.

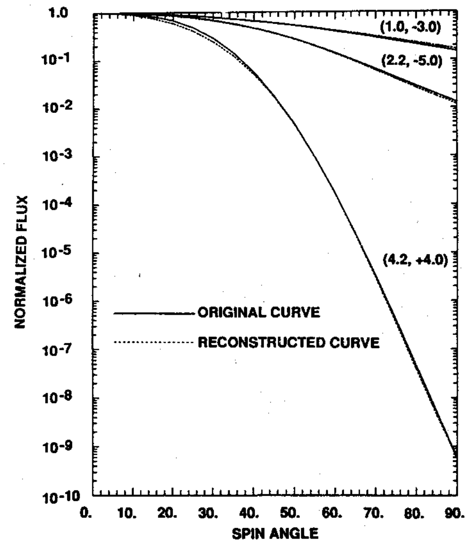

Fig. 4 Normalized model flux (dotted) and normalized original flux (solid) as functions of spin angle; parameter values are given in parentheses with Mach number first, followed by normalized potential.

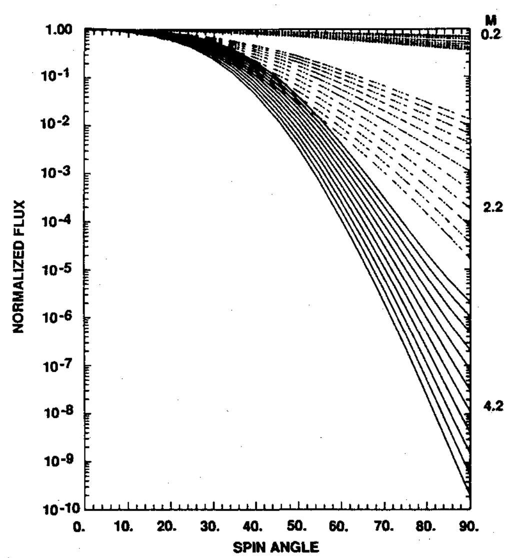

Fig. 5 Normalized flux as functions of spin angle; each group of curves corresponds to the Mach number indicated in the right margin nearest it.

viewed in this way, even these maximal differences are seen to be quite small in the context of the overall range of variation of the spin modulations (up to 10 orders of magnitude).

Spin Modulations and Discussion

With the above expressions, it is straightforward to generate curves of the spin modulation of RPA fluxes, given values for the Mach number and normalized spacecraft potential. It is worthwhile to examine the sensitivity of this modulation to these parameters. Figure 5 shows the variations of (normalized) flux into an RPA as a function of spin angle for four values of Mach number, with V_0 running the range from -5.0 (upper curve) to $+5.0$ (lower curve) (11 values) for each case. From this figure it is clear that Mach number dominates the behavior. At low Mach numbers, the curves remain tightly clustered, whereas the range of variations associated with normalized potential increases with Mach number. However, even at the highest Mach number shown, the maximum variation for the given range of potentials covers only about four orders of magnitude at 90 deg while the range of variation for these Mach numbers is 10 orders of magnitude. We also note that for each Mach number the lower curves, corresponding to the positive potentials, show greater dispersion at 90 deg than do the upper curves (negative potentials).

We can see this in a different way by plotting families of curves at constant potential with M as a parameter, as shown in Figs. 6a–6c. The Mach numbers extend from 0.2 to 4.2, as above. The main difference among the three is the increasing range of variation covered by the given range of M at 90 deg. In Fig. 6a, this range is six orders of magnitude; in Fig. 6b, eight orders of magnitude; and in Fig. 6c, ten orders of magnitude. So in addition to the sensitivity of the spin modulation increasing with Mach number, as is apparent in Fig. 5, here we see more clearly that it also increases with more positive spacecraft potentials, although not to the same degree.

Although it is difficult to make a case for comparing variations associated with a unit change in Mach number with a unit change in normalized potential, we can state that the given ranges are characteristic of the Earth's plasmasphere. Therefore, we can conclude that for plasmaspheric conditions, the spin modulation of RPA fluxes will vary more sensitively with Mach number than with spacecraft potential. Similar conclusions were reached by Samir et al.[6] from analysis of wake flux observations.

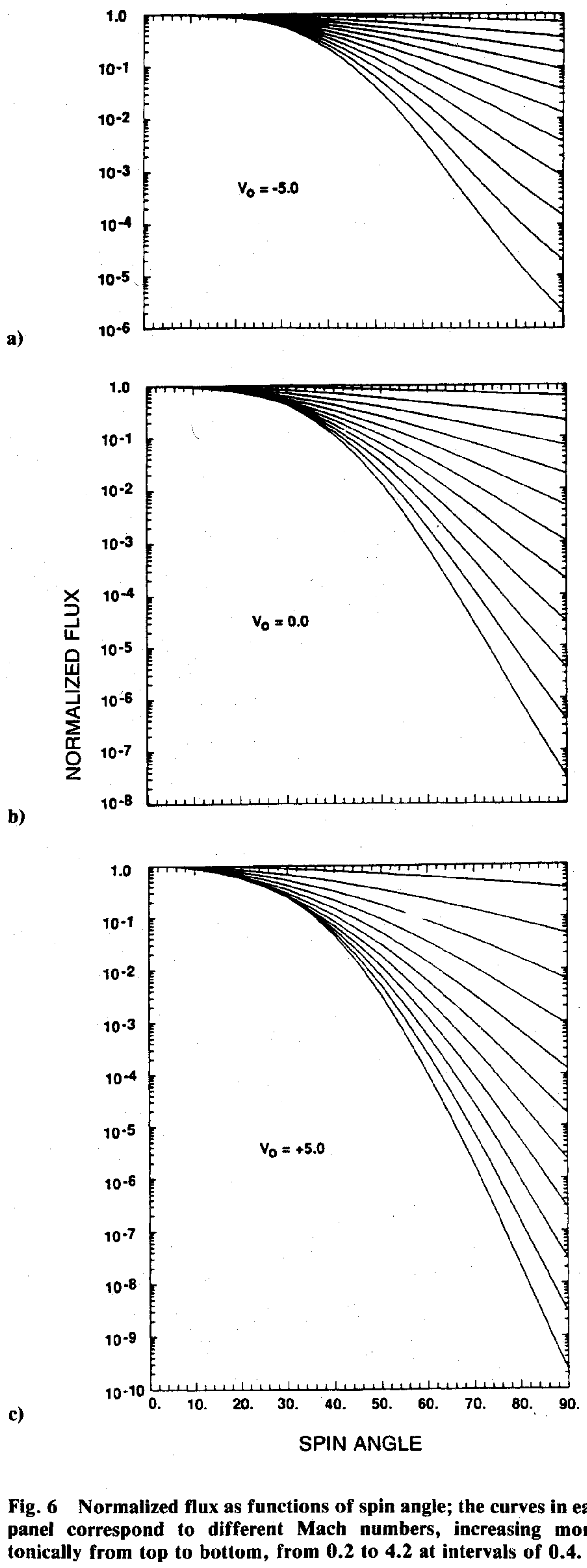

Fig. 6 Normalized flux as functions of spin angle; the curves in each panel correspond to different Mach numbers, increasing monotonically from top to bottom, from 0.2 to 4.2 at intervals of 0.4.

A similar, but more pronounced, contrast between the sensitivity to Mach number and normalized potential was found for the response of flux to the retarding potential. In that case, shape of the RPA curve is quite sensitive to Mach number while the normalized potential translates it (the entire curve) in energy.[2] In testing for convergence of the analysis technique to the correct pair of values of Mach number and spacecraft potential, Comfort et al.[2] found that the convergence for Mach number was always rapid and accurate while that for spacecraft potential was accurate primarily for negative potentials.

The variations shown in Figs. 5 and 6 suggest that the spin curve would be a useful supplement to the RPA curve in analyzing RPA data, particularly data from a positively charged spacecraft. For such a spacecraft, Comfort et al.[2] found that the convergence to a unique positive potential based on the RPA curve is intrinsically slow because the dependence is so weak. A combination of RPA analysis and spin curve analysis, two basically independent sets of information, should determine uniquely the Mach number and spacecraft potential. Such a technique has been developed and will be described in a subsequent publication.

This difference in sensitivity of the spin modulation to the Mach number and normalized spacecraft potential is what facilitated the relatively straightforward decomposition of these two dependences. Because this is a physical effect, independent of a particular spacecraft or instrument, the qualitative behavior of the spin modulation should vary with Mach number and spacecraft potential in the same way from spacecraft to spacecraft. The procedure developed here should be generally applicable to other instruments on other spacecraft, although the detailed fit parameters (as given in Table 2) would require tailoring to the specific instrument.

Limitations of this model should be kept in mind and observed in applications. These limitations are defined basically by the parameter range used to determine the generating coefficients. It should be emphasized that this is not a wake model; only spin angles between zero and 90 deg were employed in the original data set. A primary reason for this is that the numerical model on which that data set was based[1] is not an adequate wake model.[6] In addition, the range of Mach numbers and normalized potentials is also restricted, and it is not known how accurate this model is beyond this range. From experience in pushing the model beyond these limits, we have found that when extended too far, the resulting spin curves become unphysical, such that with increasing spin angle, the curves reach a minimum and begin increasing again, well before 90 deg is attained.

If the parameter range is too limited, or a different instrument is to be modeled, a new data set can be generated and a new set of coefficients determined for the conditions required, according to the procedure demonstrated above, although this may require increasingly complex polynomials to fit the coefficients. This was found to be the case in the present study. Initial ranges of the fitting parameters were Mach numbers from 0 to 2 and normalized potentials from -1 to $+2$. For these ranges, second-order polynomials give excellent fits (to better than 5% accuracy) to the coefficients, whereas for the present ranges, third-order polynomials are required for acceptable fits.

Conclusions

A technique has been presented for developing an analytical model of the spin modulation of fluxes into a retarding potential analyzer. The model is theoretical in the sense that it is based on a theoretical numerical model of the way a charged spacecraft affects the plasma in its vicinity. It is empirical in the sense that it is presented in terms of convenient analytical functions that have no intrinsic relationship to the physical problem but do provide an accurate representation. This technique is employed to provide a detailed model for the spin modulation of the ion flux to the radial detector head of the retarding ion mass spectrometer on the DE-1 spacecraft.

The accuracy of this empirical model has been examined by generating curves corresponding to those originally employed to develop the model and by comparing with those original curves. These curves have been found to represent observed ion spin-modulated fluxes well.[4] It is found that over most of the angle range 0 to 90 deg, the model is accurate to within

5%, with maximum deviations near 30 and 80 deg up to 16%. Typical spin modulations show that negative spacecraft potentials have little effect on the spin curves. However, the Mach number and positive potential variations can cause dramatic changes in the spin modulation, with the former being more important. This hierarchy of dependencies of the spin modulation on Mach number and spacecraft potential is a general result, allowing a step-by-step decomposition of the parametric dependencies to proceed in a straightforward manner, as demonstrated with the empirical model developed here. The procedure has general applicability although the coefficients in the model are specific to the DE-1/RIMS instrument. It appears that use of this model in analyzing spin-modulated ion data, in combination with the analysis of RPA data, would be very helpful in obtaining unique values for the Mach number and spacecraft potential, including data from positively charged spacecraft.

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